Bulg. J. Phys. **44** (2017) 276–287

Inhomogeneous Generalization of Bianchi Type VI_h String Cosmological Model for Stiff Perfect Fluid Distribution

A. Sharma[1], **A. Tyagi**[2], **D. Chhajed**[1]

[1]Department of Mathematics, Sir Padampat Singhania University, Udaipur–313601, Rajasthan, India

[2]Department of Mathematics and Statistics, University College of Science, MLS University, Udaipur–313001, Rajasthan, India

Received *25 November 2016*

Abstract. We have investigated inhomogeneous generalization of Bianchi type VI_h string cosmological model for stiff perfect fluid distribution. To obtain the deterministic solution of Einstein's field equations, we assume that the isotropic pressure p is equal to string rest density ρ. Each of the cases $1 + h = 0$ and $1 + h \neq 0$ gives rise to families of universe. The model obtained is expanding, shearing and non-rotating universe. Some physical and geometrical features of the model are also discussed.

PACS codes: 98.80.jk; 04.20-q.Ex; 04.60.Cf

1 Introduction

Present cosmology is very well defined by various Bianchi type cosmological models, which are completely homogeneous and anisotropic, but Maccallum [1], Collins and Hawking [2] have emphasized that at early stage of its evolution, the universe may have been highly inhomogeneous and anisotropic. It could become homogeneous and isotropic through the cosmological expansion. So construction of exact inhomogeneous generalization of certain Bianchi type cosmological models is the main issue to study the process of homogenization and isotropization of the universe. Remarkable work has been done in obtaining various generalize inhomogeneous Bianchi cosmological models. Belinskii et al. [3] and Barrow and Tippler [4]. Wainwright et al. [5] have obtained some exact solutions, which generalize Bianchi type III, V and VI_h models for vacuum and for stiff perfect fluid. Carmeli et al. [6] have constructed new inhomogeneous generalizations of Bianchi type III, V and VI_h models for vacuum and for the case in which mass less scalar field is present. Roy and Narain [7] have derived solutions, which generalize Bianchi type VI_0 cosmological models for perfect fluid distribution. Roy and Prasad [8] have obtained solutions, which generalize the Bianchi type VI_h cosmological models with perfect fluid. Roy and Prasad [9] have also derived solutions generalizing the Bianchi type VI_h cosmological models with stiff perfect fluid and radiation.

The large scale distribution of galaxies in our universe show that the material distribution can satisfactorily distributed by perfect fluid. It is, however, hypothesized that universe might have undergone a series of phase transition after a big bang explosion. These phase transition produce vacuum domain wall, strings and monopoles. Among these, cosmic strings play a significant interest as these act gravitational lenses which give rise to density perturbation leading to the formation of galaxies. The general relativistic formalism of cosmic string is due to Latelier [10, 11]. Stachel [12] has developed massless string. Tyagi et al. [13] have obtained the solutions of field equations for inhomogeneous Bianchi type VI_0 string dust cosmological model of perfect fluid distribution. So far a considerable amount of work has been done on cosmic strings and string cosmological models by Krori et al. [14, 15], Tikekar and Patel [16], Bali and Singh [17].

In this paper, we have investigated inhomogeneous generalization of Bianchi type VI_h string cosmological model for stiff perfect fluid distribution. For the complete deterministic solution of the Einstein's field equations, we assume that the isotropic pressure p is equal to string rest density ρ. We have studied for: (i) $1 + h \neq 0$, $\beta_4 \neq 0$ (but any constant k); (ii) $1 + h \neq 0$, $\beta_4 = 0$; and (iii) $1 + h = 0$, $\beta_4 \neq 0$. The various physical and geometrical aspects of the models are also discussed.

2 Solution of the Field Equations

We take the line element in the form

$$ds^2 = e^{2\alpha}(dx^2 - dt^2) + e^{\beta+\gamma+2x}dy^2 + e^{\beta-\gamma+2hx}dz^2\,, \tag{1}$$

where $\alpha = \alpha(x,t)$, $\beta = \beta(t)$, and $\gamma = \gamma(t)$, h being constant. The universe described by the line element (1) is filled with co-moving string perfect fluid satisfying the Einstein's field equations (in gravitational units $8\pi G = c = 1$).

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j = -\left[(\rho+p)v_iv^j + pg_i^j - \lambda x_ix^j\right], \tag{2}$$

where T_i^j is the energy-momentum tensor for a cloud of massive strings and perfect fluid distribution, v_i is a unit flow vector and x_i satisfies the conditions $v_iv^i = -x_ix^i = -1$, $v^ix_i = 0$.

Here ρ is the rest energy of the cloud of strings with massive particles attached to them, p is the pressure and λ – the density of tension that characterizes the strings. The unit space-like vector x^i represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector v^i describes the four-velocity vector of the matter satisfying the following conditions $g_{ij}v^iv^j = -1$. In the present scenario, the co-moving coordinates are taken as

$$v^i = (0, 0, 0, e^{-\alpha}) \,, \tag{3}$$

and choose x^i parallel to x-axis so that

$$x^i = (e^{-\alpha}, 0, 0, 0) \,. \tag{4}$$

The Einstein field equations (2) for the line element (1) lead to the following system of equations:

$$e^{-2\alpha}\left[\beta_{44} + \frac{3}{4}\beta_4^2 + \frac{1}{4}\gamma_4^2 - \alpha_4\beta_4 - (1+h)\alpha_1 - h\right] = -p + \lambda \,, \tag{5}$$

$$e^{-2\alpha}\left[\alpha_{44} - \alpha_{11} + \frac{1}{2}(\beta_{44} - \gamma_{44}) + \frac{1}{4}(\beta_4 - \gamma_4)^2 - h^2\right] = -p \,, \tag{6}$$

$$e^{-2\alpha}\left[\alpha_{44} - \alpha_{11} + \frac{1}{2}(\beta_{44} + \gamma_{44}) + \frac{1}{4}(\beta_4 + \gamma_4)^2 - 1\right] = -p \,, \tag{7}$$

$$e^{-2\alpha}\left[\frac{1}{4}(\beta_4^2 - \gamma_4^2) + \alpha_4\beta_4 + (1+h)\alpha_1 - (1+h+h^2)\right] = \rho \,. \tag{8}$$

$$\alpha_1\beta_4 + (1+h)\alpha_4 - \frac{1}{2}(1+h)\beta_4 - \frac{1}{2}(1-h)\gamma_4 = 0 \,. \tag{9}$$

On subtracting equation (6) from (7), we get

$$\gamma_{44} + \beta_4\gamma_4 = (1 - h^2) \,. \tag{10}$$

On adding equations (6) and (7), we get

$$e^{-2\alpha}\left[2\alpha_{44} - 2\alpha_{11} + \beta_{44} + \frac{1}{2}\beta_4^2 + \frac{1}{2}\gamma_4^2 - 1 - h^2\right] = -2p \,. \tag{11}$$

Equation (8) together with the stiff fluid equation of state, i.e. $\rho = p$, imply

$$e^{-2\alpha}\Big[\frac{1}{2}(\beta_4^2 - \gamma_4^2) + 2\alpha_4\beta_4 + 2(1+h)\alpha_1 - 2(1+h+h^2)\Big] = 2p \,. \tag{12}$$

From equations (11) and (12), we get

$$\Big[2\alpha_{44} - 2\alpha_{11} + \beta_{44} + \beta_4^2 + 2\alpha_4\beta_4 + 2(1+h)\alpha_1 - (3 + 2h + 3h^2)\Big] = 0, \tag{13}$$

The three equations (9), (10) and (13) are in three unknowns α, β, and γ. We solve them for the following three different cases.

CASE I: $1+h\neq 0$ and but any constant k.
Equation (9) implies

$$\alpha=\phi+\frac{1}{2}\beta+\frac{1}{2}\Big(\frac{1-h}{1+h}\Big)\gamma\,, \tag{14}$$

where $\alpha=\phi[(1+h)x-\beta]$. Hence from equations (10), (13) and (14), we get

$$\left[k^2-(1+h)^2\right](\phi''-\phi'+1)=0\,.$$

If $[k^2-(1+h)^2]\neq 0$, then

$$\phi''-\phi'+1=0\,, \tag{15}$$

where a prime $(')$ denotes differentiation w.r.t. $[(1+h)x-\beta]$.
Equation (15) gives

$$\phi=n_1+n_2e^{[(1+h)x-\beta]}+[(1+h)x-\beta]\,, \tag{16}$$

where n_1 and n_2 are constants of integration.
Now taking $\beta_4=k$ on integration, we get

$$\beta=kt+k_1\,, \tag{17}$$

where k_1 is constant of integration.
Equation (10) gives on integration

$$\gamma=c_1+c_2e^{-kt}+\frac{(1-h^2)}{k}t\,, \tag{18}$$

where c_1 and c_2 are constants of integration.

$$\phi=n_1+n_2e^{[(1+h)x-(kt+k_1)]}+[(1+h)x-(kt+k_1)]\,. \tag{19}$$

Therefore,

$$\alpha=L_1e^{[(1+h)x-kt]}+L_2e^{-kt}+L_3t+(1+h)x+L_4\,, \tag{20}$$

where

$$L_1=n_2e^{-k}\,;\qquad L_2=\frac{1}{2}\frac{(1-h)}{(1+h)}c_2\,;$$

$$L_3=\frac{(1-h)^2-k^2}{2k}\,;\quad\text{and}\quad L_4=n_1-\frac{k}{2}+\frac{1}{2}\frac{(1-h)}{(1+h)}c_1$$

are new constants.
So the line element (1) becomes in this case

$$ds^2 = \exp\Big\{2\big[L_1 e^{[(1+h)x-kt]} + L_2 e^{-kt} + L_3 t + (1+h)x + L_4\big]\Big\} \times (dx^2 - dt^2) + \exp[c_2 e^{-kt} + (L_5 + k)t + L_6 + 2x]dy^2 + \exp[-c_2 e^{-kt} + (k - L_5)t + L_7 + 2hx]dz^2\,, \quad (21)$$

where $L_5 = \dfrac{(1-h^2)}{k}$, $L_6 = k + c_1$ and $L_7 = k - c_1$.

CASE II: $1 + h \neq 0$ and $\beta_4 = 0$.

Equation (10) gives on integration

$$\gamma = (1-h^2)\frac{t^2}{2} + l_1 t + l_2\,. \quad (22)$$

Taking now $\beta_4 = 0$ on integration we get

$$\beta = l \quad (23)$$

and hence equation (9) gives

$$\alpha = (1-h)^2\frac{t^2}{4} + \frac{1}{2}\frac{(1-h)}{(1+h)}l_1 t + L\,, \quad (24)$$

L being arbitrary functions of x only and l, l_1 and l_2 are constants of integration. Equation (13) determines L as

$$L = m_1 + m_2 e^{(1+h)x} + (1+h)x\,, \quad (25)$$

where m_1 and m_2 are constants of integration.

Therefore,

$$\alpha = (1-h)^2\frac{t^2}{4} + \frac{1}{2}\frac{(1-h)}{(1+h)}l_1 t + m_1 + m_2 e^{(1+h)x} + (1+h)x\,. \quad (26)$$

So the line element (1) becomes in this case

$$ds^2 = \exp 2\Big\{(1-h)^2\frac{t^2}{4} + \frac{1}{2}\Big(\frac{1-h}{1+h}\Big)l_1 t + m_1 + m_2 e^{(1+h)x} + (1+h)x\Big\} \times (dx^2 - dt^2) + \exp\Big\{(1-h^2)\frac{t^2}{2} + l_1 t + l_2 + l + 2x\Big\}dy^2 + \exp\Big\{-(1-h^2)\frac{t^2}{2} - l_1 t - l_2 + l + 2hx\Big\}dz^2\,. \quad (27)$$

CASE III: $1 + h = 0$ and $\beta_4 \neq 0$.

Equation (9) implies

$$\alpha = \frac{\gamma_4}{\beta_4} x + Q(t)\,, \tag{28}$$

where Q is an arbitrary function of t; whereas equation (10) implies

$$\gamma_4 = s_1 e^{-\beta}\,, \tag{29}$$

where s_1 is a constant of integration.

From equations (13), (28) and (29), we get

$$x\Big[\frac{d^2}{dt^2}\Big(\frac{\gamma_4}{\beta_4}\Big) + \beta_4 \frac{d}{dt}\Big(\frac{\gamma_4}{\beta_4}\Big)\Big] + \Big[\frac{d^2 Q}{dt^2} + \beta_4 \frac{dQ}{dt} - 2\Big] + \frac{\beta_{44} + \beta_4^2}{2} = 0\,, \tag{30}$$

This implies that

$$\frac{d^2}{dt^2}\left(\frac{\gamma_4}{\beta_4}\right) + \beta_4 \frac{d}{dt}\left(\frac{\gamma_4}{\beta_4}\right) = 0, \tag{31}$$

From equations (29) and (31), we get

$$\beta_4 = \frac{s_1}{a_1} \exp\Big[-\Big(1 + \frac{a_1}{s_1}\Big)\beta\Big], \tag{32}$$

$$\exp\beta = \Big[\frac{s_1 + a_1}{a_2} t + a_4\Big]^{s_1/(s_1+a_1)}\,, \tag{33}$$

and

$$\gamma = \frac{a_2 s_1}{a_1} \exp\Big(\frac{a_1}{s_1}\beta\Big) + a_3\,. \tag{34}$$

Again from (30) which implies that

$$\Big[\frac{d^2 Q}{dt^2} + \beta_4 \frac{dQ}{dt} - 2\Big] + \frac{\beta_{44} + \beta_4^2}{2} = 0\,. \tag{35}$$

From equations (33) and (35), we get

$$\begin{aligned} Q = \frac{2a_2}{2s_1 + a_1}\Big[\Big(\frac{s_1 + a_1}{a_2}\Big)\frac{t^2}{2} + a_4 t\Big] \\ - \frac{1}{2}\Big(\frac{s_1}{s_1 + a_1}\Big) \log\Big[\Big(\frac{s_1 + a_1}{a_2}\Big)t + a_4\Big] \\ + \frac{a_2 a_5}{a_1}\Big[\Big(\frac{s_1 + a_1}{a_2}\Big)t + a_4\Big]^{a_1/(s_1+a_1)} + a_6\,. \end{aligned} \tag{36}$$

From equations (28), (29), (32), (33) and (36), we get

$$\alpha = \Big(a_2 + \frac{a_2 a_5}{a_1}\Big)\Big[\Big(\frac{s_1+a_1}{a_2}\Big)t + a_4\Big]^{a_1/(s_1+a_1)} + \Big(\frac{2a_2}{2s_1+a_1}\Big)\Big[\Big(\frac{s_1+a_1}{a_2}\Big)\frac{t^2}{2} + a_4 t\Big] - \frac{1}{2}\Big(\frac{s_1}{s_1+a_1}\Big)\log\Big[\Big(\frac{s_1+a_1}{a_2}\Big)t + a_4\Big] + a_6 . \tag{37}$$

After using suitable coordinate transformations and renaming of constants, the line element (1) reduces to the form

$$ds^2 = Bt^{-\frac{s_1}{s_1+a_1}} \exp\Big[2(Cx+C')t^{\frac{a_1}{s_1+a_1}} + 2\Big(\frac{s_1+a_1}{2s_1+a_1}\Big)t^2\Big](dx^2 - dt^2) + t^{\frac{s_1}{s_1+a_1}} \exp\Big(C'' t^{\frac{a_1}{s_1+a_1}} + 2x\Big)dy^2 + t^{\frac{s_1}{s_1+a_1}} \exp\Big(-C'' t^{\frac{a_1}{s_1+a_1}} + 2hx\Big)dz^2 , \tag{38}$$

where

$$B = \Big(\frac{s_1+a_1}{a_2}\Big)^{-\frac{s_1}{s_1+a_1}} \exp\Big\{2a_6 - \frac{2a_2^2 a_4^2}{(s_1+a_1)(2s_1+a_1)}\Big\},$$

$$C = \Big(\frac{s_1+a_1}{a_2}\Big)^{\frac{a_1}{s_1+a_1}} a_2 ,$$

$$C' = \frac{a_5 a_2}{a_1}\Big(\frac{s_1+a_1}{a_2}\Big)^{\frac{a_1}{s_1+a_1}} , \quad \text{and}$$

$$C'' = \frac{a_2 s_1}{a_1}\Big(\frac{s_1+a_1}{a_2}\Big)^{\frac{a_1}{s_1+a_1}}$$

are new constants.

3 Physical and Geometrical Properties

The physical and geometrical properties of the model are given as follows:

Case I: Magnitude of rotation ω is zero, i.e

$$\omega = 0 . \tag{39}$$

The expansion scalar θ of the model is given by

$$\theta = \frac{(-kL_1)e^{(1+h)x-kt} - kL_2 e^{-kt} + L_3 + k}{\exp[L_1 e^{(1+h)x-kt} + L_2 e^{-kt} + L_3 t + (1+h)x + L_4]} . \tag{40}$$

The shear σ of the model is given by

$$\sigma = \frac{\sqrt{\{2((-kL_1)e^{(1+h)x-kt} - kL_2 e^{-kt} + L_3) - k\}^2 + 3\{-c_2 k e^{-kt} + L_5\}^2}}{2\sqrt{3}\exp[L_1 e^{(1+h)x-kt} + L_2 e^{-kt} + L_3 t + (1+h)x + L_4]} . \tag{41}$$

The pressure p and the rest density ρ of the model are given by

$$p = \rho = \frac{e^{(1+h)x-kt} L_1\{(1+h)^2 - k^2\} - L_2 k^2 e^{-kt} - \frac{1}{4}(-c_2 k e^{-kt} + L_5)^2 + L_8}{\exp[2\{L_1 e^{(1+h)x-kt} + L_2 e^{-kt} + L_3 t + (1+h)x + L_4\}]}, \tag{42}$$

where $L_8 = \dfrac{k^2}{4} + L_3 + h$.

The string tension density λ of the model is given by

$$\lambda = \frac{k^2 - (1+h)^2}{\exp[2(L_1 e^{(1+h)x-kt} + L_2 e^{-kt} + L_3 t + (1+h)x + L_4)]} \tag{43}$$

The deceleration parameter q of the model is given by

$$q = -1 - 3e^{\alpha} \frac{e^{kt}\left[2(L_1 e^{(1+h)x} + L_2) - \frac{L_3}{k} e^{kt}\right] - \left(L_1 e^{(1+h)x} + L_2 - \frac{L_3}{k} e^{kt}\right)^2}{\left[L_1 e^{(1+h)x} + L_2 - (\frac{L_3}{k} - 1) e^{kt}\right]^2}. \tag{44}$$

Case II: The magnitude of rotation ω is zero, i.e.

$$\omega = 0\,. \tag{45}$$

The expansion scalar θ of the model is given by

$$\theta = \frac{(1-h)^2 \dfrac{t}{2} + \dfrac{1}{2}\Big(\dfrac{1-h}{1+h}\Big) l_1}{\exp\left[(1-h)^2 \dfrac{t^2}{4} + \dfrac{1}{2}\Big(\dfrac{1-h}{1+h}\Big) l_1 t + m_1 + m_2 e^{(1+h)x} + (1+h)x\right]}. \tag{46}$$

The shear σ of the model is given by

$$\sigma = \frac{\sqrt{\left[(1-h)^2 t + \Big(\dfrac{1-h}{1+h}\Big) l_1\right]^2 + 3\Big[(1-h^2)t + l_1\Big]^2}}{2\sqrt{3}\exp\left[(1-h)^2 \dfrac{t^2}{4} + \dfrac{1}{2}\Big(\dfrac{1-h}{1+h}\Big) l_1 t + m_1 + m_2 e^{(1+h)x} + (1+h)x\right]}. \tag{47}$$

The pressure p and the rest density ρ of the model are given by

$$p = \rho = \frac{m_2 (1+h)^2 e^{(1+h)x} - \frac{1}{4}[(1-h^2)t + l_1]^2 + h}{\exp\left\{(1-h)^2 \dfrac{t^2}{2} + \Big(\dfrac{1-h}{1+h}\Big) l_1 t + 2m_1 + 2m_2 e^{(1+h)x} + 2(1+h)x\right\}}. \tag{48}$$

The string tension density λ of the model is given by

$$\lambda = \frac{-(1+h)^2}{\exp\Big\{(1-h)^2\frac{t^2}{2}+\Big(\frac{1-h}{1+h}\Big)l_1 t+2m_1+2m_2 e^{(1+h)x}+2(1+h)x\Big\}}\,. \tag{49}$$

The deceleration parameter q of the model is given by

$$q = -1 - 3e^{\alpha}\Big[\frac{2(1+h)^2}{\{(1-h^2)t+l_1\}^2} - 1\Big]. \tag{50}$$

Case III: The magnitude of rotation ω is zero, i.e

$$\omega = 0\,. \tag{51}$$

The expansion scalar θ of the model is given by

$$\theta = \frac{(R_1 x + R_1^{'})t^{-\frac{s_1}{s_1+a_1}} + R_2 t + R_3 t^{-1}}{\sqrt{B}t^{\frac{-s_1}{2(s_1+a_1)}}\exp\Big\{(Cx+C')t^{\frac{a_1}{s_1+a_1}} + R_2 t^2\Big\}}\,, \tag{52}$$

where

$$\begin{aligned}
R_1 &= a_1\Big(\frac{s_1+a_1}{a_2}\Big)^{-s_1/(s_1+a_1)}\,,\\
R_1' &= a_5\Big(\frac{s_1+a_1}{a_2}\Big)^{-s_1/(s_1+a_1)}\,,\\
R_2 &= \frac{s_1+a_1}{2s_1+a_1}\,, \quad \text{and}\\
R_3 &= \frac{s_1}{2(s_1+a_1)}\,.
\end{aligned}$$

The shear σ of the model is given by

$$\sigma = \frac{\sqrt{4\Big[(R_1x+R_1')t^{-\frac{s_1}{s_1+a_1}}+R_2t-2R_3t^{-1}\Big]^2+3\Big[R_4t^{-\frac{s_1}{s_1+a_1}}\Big]^2}}{2\sqrt{3}\,\sqrt{B}\,t^{\frac{-s_1}{2(s_1+a_1)}}\exp\Big[(Cx+C')t^{\frac{a_1}{s_1+a_1}}+R_2t^2\Big]}\,. \tag{53}$$

The pressure p and the rest density ρ of the model are given by

$$\begin{aligned}
\rho &= p\\
&= \frac{2R_3(R_1x+R_1')t^{-\frac{2s_1+a_1}{s_1+a_1}}-R_3^2t^{-2}-\frac{1}{4}R_4^2t^{-\frac{2s_1}{s_1+a_1}}+R_2R_3-1}{Bt^{\frac{-s_1}{s_1+a_1}}\exp\Big[2(Cx+C')t^{\frac{a_1}{s_1+a_1}}+2R_2t^2\Big]}\,.
\end{aligned} \tag{54}$$

The string tension density λ of the model is given by

$$\lambda = \frac{2R_3(2R_3-1)}{Bt^{\frac{-s_1}{s_1+a_1}+2}\exp\left[2(Cx+C')t^{\frac{a_1}{s_1+a_1}}+2R_2t^2\right]}. \tag{55}$$

The deceleration parameter q of the model is given by

$$q = -1-3e^{\alpha}\left[\frac{(R_1x+R_1')\left(\frac{-s_1}{s_1+a_1}\right)t^{-\frac{2s_1+a_1}{s_1+a_1}}+2R_2-R_3t^{-2}}{\left[(R_1x+R_1')t^{-\frac{s_1}{s_1+a_1}}+2R_2t+R_3t^{-1}\right]^2}\right.$$
$$\left. -\frac{(R_1x+R_1')t^{-\frac{s_1}{s_1+a_1}}+2R_2T-R_3T^{-1}}{(R_1x+R_1')T^{-\frac{s_1}{s_1+a_1}}+2R_2t+R_3t^{-1}}\right]. \tag{56}$$

4 Conclusion

We have investigated inhomogeneous generalization of Bianchi type VI_h string cosmological model for stiff fluid. We have studied for: (i) $1+h\neq 0$, $\beta_4\neq 0$ (but any constant k); (ii) $1+h\neq 0$, $\beta_4=0$; and (iii) $1+h=0$, $\beta_4\neq 0$.

In Case I, the model (21) starts expanding at $t=0$. The expansion θ in the model increases as time increases, i.e. $\theta\to\infty$ as $t\to\infty$, when $k>0$ and $L_3<0$, however the expansion θ in the model decreases as time increases and expansion stops at $t=\infty$, when $k>0$ and $L_3>0$. For large values of t, the ratio of the shear σ and expansion θ tends to a finite value, i.e.

$$\frac{\sigma}{\theta} = \frac{\left[(2L_3-k)^2+3L_5^2\right]^{\frac{1}{2}}}{2\sqrt{3}\,(L_3+k)}$$

is a constant. Hence, the model does not approach isotropy for large values of t. The fluid flow is irrotational, and it is observed that the energy density ρ and the string tension density λ tend to constant values as $t\to 0$ and $x\to 0$. At a later stage both ρ and λ approach zero when $t\to\infty$ and $x\to\infty$ as expected. Therefore, the string will disappear from the universe at a later time. As $t\to 0$ and $x\to 0$, the deceleration parameter, q, approaches to -1 when $L_1=-L_2$ and $L_3=0$ as in de Sitter universe. Therefore, the model describes an accelerating phase of the universe. In general the model represents expanding, shearing and non-rotating universe.

In Case II, the model (27) starts expanding at $t=0$ and as $t\to\infty$, θ becomes zero, i.e. the expansion stops. For large values of t, the ratio of the shear σ and the expansion θ tends to a finite value, i.e.

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\left[1+3\left(\frac{1+h}{1-h}\right)^2\right],$$

where $1 - h \neq 0$. Hence, the model does not approach isotropy for large values of t. The fluid flow is irrotational, and it is observed that the energy density ρ and the string tension density λ tend to constant values as $t \to 0$ and $x \to 0$. At a later stage both ρ and λ approach zero when $t \to \infty$ and $x \to \infty$ as expected. Therefore, the string will disappear from the universe at a later time. As $t \to 0$, $x \to 0$, deceleration parameter q approaches 0 when $m_1 = -m_2$ and $h = \pm\frac{l}{\sqrt{3}} - 1$, i.e., the universe expands at a constant rate.

When $m_1 = -m_2$ and $\frac{(1+h)^2}{l^2} < \frac{1}{3}$, the parameter q approaches a value, greater than zero. Therefore, the model presents a decelerating phase of the universe;

$q < -1$ when $m_1 = -m_2$ and $\frac{(1+h)^2}{l^2} > \frac{1}{2}$, therefore, the model suggests super-exponential expansion of the universe.

The parameter $q = -1$ when $m_1 = -m_2$ and $h = \pm\frac{l}{\sqrt{2}} - 1$, i.e. the model represents exponential expansion of the universe (also known as de Sitter expansion). In general the model represents expanding, shearing and non-rotating universe.

In Case-III, the model (38) starts with a big bang at $t = 0$ when $\frac{s_1}{s_1 + a_1} < 1$ and goes on expanding till $t = \infty$. When $\frac{s_1}{s_1 + a_1} < 0$ and $t \to \infty$, θ becomes zero. It is clear that as t increases, the ratio of the shear σ and the expansion θ tends to a finite value, i.e. $\frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}}$ as $t \to \infty$ when $\frac{s_1}{s_1 + a_1} > -1$. Hence, the model does not approach isotropy for large values of t. The fluid flow is irrotational and it is observed that $t \to 0$ and $x \to 0$, the energy density $\rho \to \infty$, when $\frac{s_1}{s_1 + a_1} < 1$ and as $t \to \infty$ and $x \to \infty$, $\rho \to 0$ when $0 < \frac{s_1}{s_1 + a_1} < 1$. As $t \to 0$ and $x \to 0$, the string tension density $\lambda \to \infty$ when $\frac{s_1}{s_1 + a_1} < 1$ and as $t \to \infty$ and $x \to \infty$, $\lambda \to 0$ when $\frac{s_1}{s_1 + a_1} < 2$, therefore, the string will disappear from the universe at a later time. The deceleration parameter q approaches to -1 as $t \to 0$ and $\frac{s_1}{s_1 + a_1} < 0$, as in de Sitter universe. The model represents accelerating universe. In general the model represents expanding, shearing and non-rotating universe.

References

[1] M. A. H. MacCallum (1971) *Nature Phys. Sci.* **230** 112.

[2] C. B. Collins and S. W. Hawking (1973) *Astrophys. J.* **180** 317.

[3] V. A. Belinskii, E. M. Lifshitz and I. M. Khalatnikov (1972) *Soviet Phys. JETP* **35** 838.

[4] J. D. Barrow and F. J. Tippler (1979) *Phys. Reports* **56(7)** 371.

[5] J. Wainwright, W. C. W. Ince and B. J. Marshman (1979) *Gen. Relativ. Gravit.* **10** 259.

[6] M. Carmeli, Ch. Charach and S. Malin (1981) *Phys. Reports* **76(2)** 79.

[7] S. R. Roy, and S. Narain (1985) *Astrophys. Space Sci.* **108** 195.

[8] S. R. Roy and A. Prasad (1991) *Astrophys. Space Sci.* **181** 161.

[9] S. R. Roy and A. Prasad (1995) *Astrophys. Space Sci.* **225** 51.

[10] P. S. Letelier (1979) *Phys. Rev. D* **20(6)** 1294.

[11] P. S. Letelier (1983) *Phys. Rev. D* **28(10)** 2414.

[12] J. Stachel (1980) *Phys. Rev. D* **21** 2171.

[13] A. Tyagi, A. Sharma and D. Chhajed (2015) *Raj. Acad. of Phys.* Sci. **14** 15.

[14] K.D. Krori, T. Choudhury, R. Mahanta and A. Mazumdar (1990) *Gen. Relativ. Gravit.* **22** 123.

[15] K.D. Krori, T. Choudhury and C.R. Mahanta (1994) *Gen. Relativ. Gravit.* **26** 265.

[16] R. Tikekar, and L.K. Patel (1992) *Gen. Relativ. Gravit.* **24** 397.

[17] R. Bali and S. Singh (2014) *Int.J.Theor.Phys.* **53** 2082.